

Theory of ultrasonic attenuation in heavy fermion compounds

M S Ojha^{**}, G C Rout^b and S N Behera^c

^aS. C. S. College (Autonomous), Puri-752 001, Orissa, India

^bCondensed Matter Physics Group, G. M. College (Autonomous), Sambalpur-768 004, Orissa, India

^cInstitute of Physics, Sachivalaya Marg, Bhubaneswar-751 005, Orissa, India

E-mail: mojha@iopb.res.in

Abstract. The mechanism of the ultrasonic attenuation in heavy fermion compounds is not yet studied clearly both experimentally and theoretically. A microscopic theoretical model is proposed here to study the attenuation in the compounds like UPt_3 , CeCu_2Si_2 , CeRu_2Si_2 , in their normal state. We consider the Periodic Anderson Model and incorporate phonon coupling to the hybridisation between the conduction electrons and f -electrons as well as to the f -electrons alone. The phonon Green's function is calculated by Zubarev technique. The temperature dependence of the ultrasonic attenuation coefficient (α) is calculated from the imaginary part of the phonon self-energy and the velocity of sound in the dynamic and long wavelength limit. The dependence of the parameters like the electron-phonon coupling parameters (g , ν), hybridisation (v), position of f -level (d), Lomb correlation (u), frequency (ω), is investigated numerically through plots.

Keywords. Ultrasonic attenuation, heavy fermions, electron phonon interaction

MS Nos. 74 25 Ld, 75 30 Mb, 63 20 Kr

Introduction

After the discovery of heavy fermion superconductivity, ultrasonic data have revealed power law dependence for attenuation coefficients for $T < T_c$ and a maximum around ~ 0.5 K for UPt_3 is observed [1-3]. The attenuation maximum occurs exactly where the elastic constant and the velocity of sound exhibit minima [4]. It is also observed that the pronounced ultrasonic attenuation peak of URu_2Si_2 coincides with the middle of the elastic constant step indicating that the maximum of α occurs at $T_c \approx 1.2$ K.

The first attenuation measurements in zero magnetic field showing Kondo effect were done by Müller *et al* [5]. A peak in attenuation was seen at 10–12 K [6]. Similar low temperature anomalies were observed in the elastic constants and the velocity of sound for compounds CeCu_6 ($T^* = 4$ K), CeCuSi_2 ($T^* = 0$ K), URu_2Si_2 ($T^* = 70$ K), CeBe_{13} ($T^* = 340$ K) [7].

Thalmeier [8] has considered Grüneisen parameter coupling of sound waves to the bands to explain the elastic anomalies in low temperature quasi-particle regime. Others have considered the strain-dependence of the hybridisation to investigate the low temperature anomaly in sound velocity of heavy fermion

systems by using a mean field approach [9]. Rout and Das [10] have calculated velocity of sound of superconducting heavy fermion systems. In this paper, a microscopic theoretical model is proposed to investigate the ultrasonic attenuation in normal state heavy fermion systems. The attenuation Müller peak [5] can be described in terms of a narrow resonance peak with width of ~ 2 meV in the density of state lying above the Fermi surface resulting from the hybridisation of f -level and the conduction band.

2. Formalism

The system is described by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{e-p} + \mathcal{H}_f, \quad (1)$$

$$\mathcal{H}_0 = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + \epsilon_f \sum_{\mathbf{k}, \sigma} f_{\mathbf{k}, \sigma}^\dagger f_{\mathbf{k}, \sigma}$$

$$+ V \sum_{\mathbf{k}, \sigma} (f_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + c_{\mathbf{k}, \sigma}^\dagger f_{\mathbf{k}, \sigma})$$

$$+ (U/2) \sum_{i, \sigma} n_{i, \sigma}^f n_{i, -\sigma}^f, \quad (2)$$

$$\mathcal{H}_{e-p} = \sum_{\kappa, q, \sigma} \left[f_1(q) \left(c_{\kappa+q, \sigma}^\dagger f_{\kappa, \sigma} + f_{\kappa+q, \sigma}^\dagger c_{\kappa, \sigma} \right) + f_2(q) \left(f_{\kappa+q, \sigma}^\dagger f_{\kappa, \sigma} \right) \right] (b_q + b_{-q}^\dagger), \quad (3)$$

$$\mathcal{H}_p = \sum_q \omega_q b_q^\dagger b_q. \quad (4)$$

\mathcal{H}_0 , \mathcal{H}_{e-p} and \mathcal{H}_p are the Hamiltonians representing Periodic Anderson Model, electron-phonon interaction and free phonon term. Here $c_{\kappa, \sigma}^\dagger$ ($c_{\kappa, \sigma}$) $f_{\kappa, \sigma}^\dagger$ ($f_{\kappa, \sigma}$), b_q^\dagger (b_q) are the creation (annihilation) operators of d - and f - electrons and phonons respectively. ϵ_k , ϵ_f and V are the conduction electron energy, position of f -level and hybridisation respectively. U is the Coulomb interaction. $f_1(q)$ and $f_2(q)$ are the phonon coupling to hybridisation and f -electron respectively.

The double time Green's function of Zubarev type is defined as

$$D_{q, q}(t-t') = -i\Theta(t-t') \langle [A_q(t); A_q(t')] \rangle. \quad (5)$$

Applying Dyson's approximation, the phonon Green function can be written as

$$D_{q, q}(\omega) = (\omega_q / \pi) [\omega^2 - \omega_q^2 - \Sigma_q(\omega)]^{-1}, \quad (6)$$

where phonon self energy is given by

$$\begin{aligned} \Sigma_q(\omega) &= 4\pi \omega_q \chi_{q, q}(\omega), \\ \chi_{q, q}(\omega) &= \sum_{\kappa, \sigma} \left[f_1^2(q) \Gamma_3 + f_1(q) f_2(q) (\Gamma_4 + \Gamma_5) + f_2^2(q) \Gamma_6 \right]. \end{aligned} \quad (7)$$

$\Gamma_i(k, q, \omega)$'s ($i = 3$ to 6) represent the electron response functions. They are defined by dropping k, q, ω in terms of α 's and β 's as

$$\begin{aligned} \Gamma_3 &= \langle\langle (\alpha^a + \alpha^b); (\beta^a + \beta^b) \rangle\rangle_\omega, \\ \Gamma_4 &= \langle\langle (\alpha^a + \alpha^b); \beta^c \rangle\rangle_\omega, \\ \Gamma_5 &= \langle\langle \alpha^c; (\beta^a + \beta^b) \rangle\rangle_\omega, \\ \Gamma_6 &= \langle\langle \alpha^c; \beta^c \rangle\rangle_\omega. \end{aligned} \quad (8)$$

These operators are written in abbreviated form as

$$\begin{aligned} \alpha^a &= c_{k-q, \sigma}^\dagger f_{k, \sigma}, \quad \alpha^b = f_{k-q, \sigma}^\dagger c_{k, \sigma}, \\ \alpha^c &= f_{k-q, \sigma}^\dagger f_{k, \sigma}, \quad \alpha^d = c_{k-q, \sigma}^\dagger c_{k, \sigma}. \end{aligned}$$

$$(\beta^a + \beta^b) = c_{k'-q', \sigma'}^\dagger f_{k', \sigma'} + f_{k'-q', \sigma'}^\dagger c_{k', \sigma'},$$

$$\beta^c = f_{k'-q', \sigma'}^\dagger f_{k', \sigma'}.$$

In the long wavelength limit, $q \rightarrow 0$. One has $\omega =$ and $\omega_0 = v_0 q$ where v_0 and v are the bare and renormal longitudinal sound velocities. The velocity of sound is given by

$$v = v_0 \left[1 + \frac{4\pi \chi(\omega, q)}{\omega_0} \right]^{1/2}$$

The ultrasonic attenuation coefficient $\alpha(\omega, T)$ for sound waves of frequency ω and at a temperature T is obtained from the imaginary part of self energy through

$$\alpha(\omega, T) = -\frac{4\pi}{\omega} \text{Im} \chi(\omega, q), \quad (9)$$

where the -ve sign indicates absorption of energy. To study ultrasonic attenuation, the parameters are made dimensionless with respect to Debye frequency ω_D .

$$\tilde{\omega} = \frac{\omega}{\omega_D}, \quad c = \frac{\omega}{\omega_f}, \quad p = \frac{\omega_0}{\omega_f}, \quad e = \frac{\eta}{\omega_f}, \quad d = \frac{\epsilon_f}{\omega_f};$$

$$u = \frac{U}{\omega_D}, \quad d_1 = d + un, \quad V = \frac{V}{\omega_D}, \quad g = \frac{N(0) f_1(0)^2}{\omega_D}$$

$$r = \frac{f_2(0)}{f_1(0)}, \quad t = \frac{kT}{\omega_f}, \quad \tilde{\omega}_f = \frac{\omega_f}{\omega_f}, \quad x = \frac{\omega_k}{\omega_f}, \quad W = \frac{W}{\omega_D}$$

3. Results and discussion

The velocity of sound is evaluated numerically under half-filled band situation. The Fermi level is taken at the middle of band with $\epsilon_F = 0$. The dimensionless parameters involve numerical calculations are the phonon coupling strength (r), the ratio (r) between the phonon coupling to hybridisation and the f -level, the position of the bare f -level (d), hybridisation Coulomb interaction (u). The temperature variation of ultrasonic attenuation of sound of the heavy fermion system shows unusually high anomaly at low temperatures for certain physical parameters. The anomaly is discussed below.

Figure 1 shows the variation of ultrasonic attenuation with temperature. As phonon coupling (g) to the hybridisation increases, the ultrasonic absorption increases and as a result peak height increases near temperature $t = 0.05$. With Debye frequency $\omega_D \approx 200\text{K}$, this corresponds to fluctuation temperature $T^* \approx 10\text{K}$. The phonon coupling increases hybridisation between f -electron and conduction band results in increase of mixed valence behaviour. When effective

Theory of ultrasonic attenuation in heavy fermion compounds

level lies below the Fermi level, the attenuation decreases with increase of phonon coupling (r) to the f -electron alone.

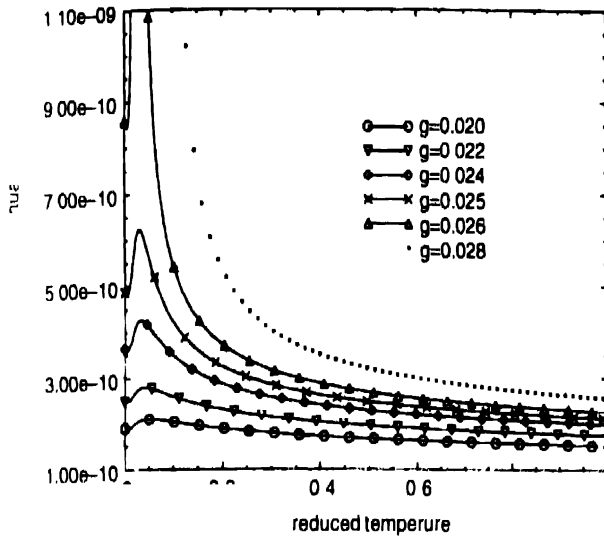


Figure 1 The variation of ultrasonic attenuation with temperature for different values of $g = 0.020, 0.022, 0.024, 0.025, 0.026, 0.028$ and fixed values of $r = 0, d = -0.94, v = 0.05, u = 1.0, e = 0.005, p = 0.12$.

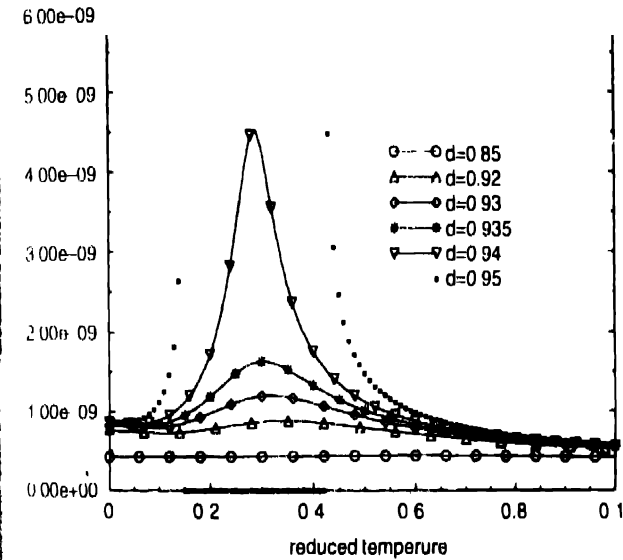


Figure 2. The variation of ultrasonic attenuation with temperature for different values of $d = -0.85, -0.92, -0.93, -0.935, -0.94, -0.95$ and fixed values of $g = 0.026, r = 0, v = 0.05, u = 1.0, e = 0.005, p = 0.12$.

Figure 2 shows the variation of attenuation peak with temperature. As f -level moves from $d = -0.85$ to -0.95 from above, towards the Fermi level $\epsilon_F = 0$, the attenuation increases and attenuation maximum shifts to lower temperature. The shifting of f -level in the conduction band changes the hybridisation between d - and f -electrons. This leads to change in the density of states near Fermi level and changes the fluctuation temperature (T^*). Hence, the ultrasonic attenuation and softening elastic constant of several heavy fermion compounds can be explained.

Acknowledgment

The authors (M.S.O. and G.C.R.) gracefully acknowledge the financial support of U.G.C., New Delhi vide letter No. F-PSO-035/99-00 (ERO) dated 08.2.2000.

References

- [1] E Bucher, B Batlogg, D J Bishop and C M Varma *J. Appl. Phys.*, **57** 3060 (1985)
- [2] V Müller, D Maurer, E W Scheidt, Ch Roth, K Lüders, E Bucher and H E Bömmel *Solid State Commun.* **7** 319 (1986)
- [3] B Golding, D J Bishop, B Batlogg, W H Haemmerle, Z Fisk and H R Ott *Phys. Rev. Lett.* **55** 2479 (1985)
- [4] C Jin, D M Lee, S W Lin, B K Sharma and D G Hinks *J. Low Temp. Phys.* **89** 557 (1992)
- [5] V Müller, D Maurer, K de Groot, E Bucher and H E Bömmel *Phys. Rev. Lett.* **56** 248 (1986)
- [6] S W Lin, I Kouroudis, A G M Jensen, P Wyder, B Luthi, D G Hinks, J B Ketterson, M Levy and B K Sharma *Physica B* **223/224** 185 (1996)
- [7] B Luthi, G Brulls, P Thalmeier, B Wolf, D Finnsterbusch and I Kouroudis *J. Low Temp. Phys.* **95** 257 (1994)
- [8] P Thalmeier *J. Phys.* **C20** 1119 (1987)
- [9] R Wojciechowski, G A Gehring and L E Major *J. Phys. Condens. Matter* **6** 9707 (1994)
- [10] G C Rout and S Das *Solid State Commun.* **109** 177 (1999)